HYDRODYNAMICS OF PUMPS

Hydrodynamics of Pumps is a reference for pump experts and a textbook for advanced students exploring pumps and pump design. This book is about the fluid dynamics of liquid turbomachines, particularly pumps. It focuses on special problems and design issues associated with the flow of liquid through a rotating machine. There are two characteristics of a liquid that lead to problems and cause a significantly different set of concerns from those in gas turbines. These are the potential for cavitation and the high density of liquids, which enhances the possibility of damaging, unsteady flows and forces. The book begins with an introduction to the subject, including cavitation, unsteady flows, and turbomachinery as well as basic pump design and performance principles. Chapter topics include flow features, cavitation parameters and inception, bubble dynamics, cavitation effects on pump performance, and unsteady flows and vibration in pumps – discussed in the three final chapters. The book is richly illustrated and includes many practical examples.

Christopher E. Brennen is Professor of Mechanical Engineering in the Faculty of Engineering and Applied Science at the California Institute of Technology. He has published more than 200 refereed articles and is especially well known for his research on cavitation and turbomachinery flows, as well as multiphase flows. He is the author of Fundamentals of Multiphase Flows and Cavitation and Bubble Dynamics and has edited several other works.
Hydrodynamics of Pumps

CHRISTOPHER E. BRENNEN

California Institute of Technology
Contents

Preface ix
Nomenclature xi

1 Introduction 1
  1.1 Subject 1
  1.2 Cavitation 1
  1.3 Unsteady Flows 2
  1.4 Trends in Hydraulic Turbomachinery 3
  1.5 Book Structure 4

2 Basic Principles 5
  2.1 Geometric Notation 5
  2.2 Cascades 8
  2.3 Flow Notation 11
  2.4 Specific Speed 12
  2.5 Pump Geometries 13
  2.6 Energy Balance 14
  2.7 Noncavitating Pump Performance 18
  2.8 Several Specific Impellers and Pumps 19

3 Two-Dimensional Performance Analysis 22
  3.1 Introduction 22
  3.2 Linear Cascade Analyses 22
  3.3 Deviation Angle 27
  3.4 Viscous Effects in Linear Cascades 28
  3.5 Radial Cascade Analyses 30
  3.6 Viscous Effects in Radial Flows 34
4 Other Flow Features
4.1 Introduction 37
4.2 Three-Dimensional Flow Effects 37
4.3 Radial Equilibrium Solution: An Example 40
4.4 Discharge Flow Management 44
4.5 Prerotation 47
4.6 Other Secondary Flows 51

5 Cavitation Parameters and Inception
5.1 Introduction 55
5.2 Cavitation Parameters 55
5.3 Cavitation Inception 58
5.4 Scaling of Cavitation Inception 62
5.5 Pump Performance 63
5.6 Types of Impeller Cavitation 65
5.7 Cavitation Inception Data 70

6 Bubble Dynamics, Damage and Noise
6.1 Introduction 78
6.2 Cavitation Bubble Dynamics 78
6.3 Cavitation Damage 83
6.4 Mechanism of Cavitation Damage 85
6.5 Cavitation Noise 88

7 Cavitation and Pump Performance
7.1 Introduction 96
7.2 Typical Pump Performance Data 96
7.3 Inducer Designs 102
7.4 Inducer Performance 104
7.5 Effects of Inducer Geometry 108
7.6 Analyses of Cavitation in Pumps 111
7.7 Thermal Effect on Pump Performance 114
7.8 Free Streamline Methods 122
7.9 Supercavitating Cascades 125
7.10 Partially Cavitating Cascades 127
7.11 Cavitation Performance Correlations 134

8 Pump Vibration
8.1 Introduction 137
8.2 Frequencies of Oscillation 140
8.3 Unsteady Flows 143
10.13 Moments and Lines of Action 246
10.14 Axial Flow Inducers 249

Bibliography 253
Index 267
Preface

This book is intended as a combination of a reference for pump experts and a monograph for advanced students interested in some of the basic problems associated with pumps. It is dedicated to my friend and colleague Allan Acosta, with whom it has been my pleasure and privilege to work for many years.

But this book has other roots as well. It began as a series of notes prepared for a short course presented by Concepts NREC and presided over by another valued colleague, David Japikse. Another friend, Yoshi Tsujimoto, read early versions of the manuscript and made many valuable suggestions.

It was a privilege to have worked on turbomachinery problems with a group of talented students at the California Institute of Technology, including Sheung-Lip Ng, David Braisted, Javier Del Valle, Greg Hoffman, Curtis Meissner, Edmund Lo, Belgacem Jery, Dimitri Chamieh, Douglas Adkins, Norbert Arndt, Ronald Franz, Mike Karyeaclis, Rusty Miskovish, Abhijit Bhattacharyya, Adiel Guinzburg, and Joseph Sivo. I recognize the many contributions they made to this book.

In the first edition, I wrote that this work would not have been possible without the encouragement, love, and companionship of my beloved wife Doreen. Since then fate has taken her from me and I dedicate this edition to our daughters, Dana and Kathy, whose support has been invaluable to me.

Christopher E. Brennen
California Institute of Technology
January 2010
### Nomenclature

**Roman letters**

- $a$: Pipe radius
- $A$: Cross-sectional area
- $A_{ijk}$: Coefficients of pump dynamic characteristics
- $[A]$: Rotordynamic force matrix
- $Ar$: Cross-sectional area ratio
- $B$: Breadth of passage or flow
- $[B]$: Rotordynamic moment matrix
- $c$: Chord of the blade or foil
- $c$: Speed of sound
- $c$: Rotordynamic coefficient: cross-coupled damping
- $c_b$: Interblade spacing
- $c_{PL}$: Specific heat of liquid
- $C$: Compliance
- $C$: Rotordynamic coefficient: direct damping
- $C_D$: Drag coefficient
- $C_L$: Lift coefficient
- $C_p$: Coefficient of pressure
- $C_{pmin}$: Minimum coefficient of pressure
- $d$: Ratio of blade thickness to blade spacing
- $D$: Impeller diameter or typical flow dimension
- $D_f$: Diffusion factor
- $D_T$: Determinant of transfer matrix $[T]$
- $e$: Specific internal energy
- $E$: Energy flux
- $E$: Young’s modulus
- $f$: Friction coefficient
- $F$: Force
- $g$: Acceleration due to gravity
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_s$</td>
<td>Component of $g$ in the $s$ direction</td>
</tr>
<tr>
<td>$h$</td>
<td>Specific enthalpy</td>
</tr>
<tr>
<td>$h$</td>
<td>Blade tip spacing</td>
</tr>
<tr>
<td>$h_p$</td>
<td>Pitch of a helix</td>
</tr>
<tr>
<td>$h^T$</td>
<td>Total specific enthalpy</td>
</tr>
<tr>
<td>$h^*$</td>
<td>Piezometric head</td>
</tr>
<tr>
<td>$H$</td>
<td>Total head rise</td>
</tr>
<tr>
<td>$H(s,\theta,t)$</td>
<td>Clearance geometry</td>
</tr>
<tr>
<td>$I$</td>
<td>Acoustic impulse</td>
</tr>
<tr>
<td>$I,J$</td>
<td>Integers such that $\omega/\Omega = I/J$</td>
</tr>
<tr>
<td>$I_P$</td>
<td>Pump impedance</td>
</tr>
<tr>
<td>$j$</td>
<td>Square root of $-1$</td>
</tr>
<tr>
<td>$k$</td>
<td>Rotordynamic coefficient: cross-coupled stiffness</td>
</tr>
<tr>
<td>$k_L$</td>
<td>Thermal conductivity of the liquid</td>
</tr>
<tr>
<td>$K$</td>
<td>Rotordynamic coefficient: direct stiffness</td>
</tr>
<tr>
<td>$K_G$</td>
<td>Gas constant</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Pipe length or distance to measuring point</td>
</tr>
<tr>
<td>$L$</td>
<td>Lift</td>
</tr>
<tr>
<td>$L$</td>
<td>Inertance</td>
</tr>
<tr>
<td>$L$</td>
<td>Axial length</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>Latent heat</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass flow rate</td>
</tr>
<tr>
<td>$m$</td>
<td>Rotordynamic coefficient: cross-coupled added mass</td>
</tr>
<tr>
<td>$m_G$</td>
<td>Mass of gas in bubble</td>
</tr>
<tr>
<td>$m_D$</td>
<td>Constant related to the drag coefficient</td>
</tr>
<tr>
<td>$m_L$</td>
<td>Constant related to the lift coefficient</td>
</tr>
<tr>
<td>$M$</td>
<td>Moment</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number, $u/c$</td>
</tr>
<tr>
<td>$M$</td>
<td>Rotordynamic coefficient: direct added mass</td>
</tr>
<tr>
<td>$n$</td>
<td>Coordinate measured normal to a surface</td>
</tr>
<tr>
<td>$N$</td>
<td>Specific speed</td>
</tr>
<tr>
<td>$N(R_N)$</td>
<td>Cavitation nuclei number density distribution function</td>
</tr>
<tr>
<td>$NPSP$</td>
<td>Net positive suction pressure</td>
</tr>
<tr>
<td>$NPSE$</td>
<td>Net positive suction energy</td>
</tr>
<tr>
<td>$NPSH$</td>
<td>Net positive suction head</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$p_A$</td>
<td>Radiated acoustic pressure</td>
</tr>
<tr>
<td>$p^T$</td>
<td>Total pressure</td>
</tr>
<tr>
<td>$p_G$</td>
<td>Partial pressure of gas</td>
</tr>
<tr>
<td>$p_S$</td>
<td>Sound pressure level</td>
</tr>
<tr>
<td>$p_V$</td>
<td>Vapor pressure</td>
</tr>
</tbody>
</table>
Nomenclature

\[ P \] Power
\[ \tilde{q}^n \] Vector of fluctuating quantities
\[ Q \] Volume flow rate (or heat)
\[ \dot{Q} \] Rate of heat addition
\[ r \] Radial coordinate in turbomachine
\[ R \] Radial dimension in turbomachine
\[ R \] Bubble radius
\[ R \] Resistance
\[ R_N \] Cavitation nucleus radius
\[ Re \] Reynolds number
\[ s \] Coordinate measured in the direction of flow
\[ s \] Solidity
\[ S \] Surface tension of the saturated vapor/liquid interface
\[ S \] Suction specific speed
\[ S_i \] Inception suction specific speed
\[ S_a \] Fractional head loss suction specific speed
\[ S_b \] Breakdown suction specific speed
\[ S_f \] Slip factor
\[ t \] Time
\[ T \] Temperature or torque
\[ T_{ij} \] Transfer matrix elements
\[ [T] \] Transfer matrix based on \( \tilde{p}T, \tilde{m} \)
\[ [T^*] \] Transfer matrix based on \( \tilde{p}, \tilde{m} \)
\[ [TP] \] Pump transfer matrix
\[ [TS] \] System transfer matrix
\[ u \] Velocity in the \( s \) or \( x \) directions
\[ u_i \] Velocity vector
\[ U \] Fluid velocity
\[ U_\infty \] Velocity of upstream uniform flow
\[ v \] Fluid velocity in non-rotating frame
\[ V \] Volume or fluid velocity
\[ w \] Fluid velocity in rotating frame
\[ \dot{W} \] Rate of work done on the fluid
\[ z \] Elevation
\[ Z_{CF} \] Common factor of \( Z_R \) and \( Z_S \)
\[ Z_R \] Number of rotor blades
\[ Z_S \] Number of stator blades

Greek letters

\[ \alpha \] Angle of incidence
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_L$</td>
<td>Thermal diffusivity of liquid</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle of relative velocity vector</td>
</tr>
<tr>
<td>$\beta_{b}$</td>
<td>Blade angle relative to cross-plane</td>
</tr>
<tr>
<td>$\gamma_n$</td>
<td>Wave propagation speed</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Geometric constant</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Deviation angle at flow discharge</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Clearance</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Angle of turn</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angular coordinate</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>Camber angle</td>
</tr>
<tr>
<td>$\theta^*$</td>
<td>Momentum thickness of a blade wake</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Thermal term in the Rayleigh-Plesset equation</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Inclination of discharge flow to the axis of rotation</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Bulk modulus of the liquid</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of fluid</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Cavitation number</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Cavitation inception number</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Fractional head loss cavitation number</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>Breakdown cavitation number</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Choked cavitation number</td>
</tr>
<tr>
<td>$\sigma_{TH}$</td>
<td>Thoma cavitation factor</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>Thermal parameter for bubble growth</td>
</tr>
<tr>
<td>$\Sigma_{1,2,3}$</td>
<td>Geometric constants</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Blade thickness</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Flow coefficient</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Head coefficient</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>Head coefficient at zero flow</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Radian frequency of whirl motion or other excitation</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Bubble natural frequency</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Radian frequency of shaft rotation</td>
</tr>
</tbody>
</table>

**Subscripts**

On any variable, $Q$:

- $Q_o$: Initial value, upstream value or reservoir value
- $Q_1$: Value at inlet
Nomenclature

\(Q_2\) Value at discharge
\(Q_a\) Component in the axial direction
\(Q_b\) Pertaining to the blade
\(Q_\infty\) Value far from the bubble or in the upstream flow
\(Q_B\) Value in the bubble
\(Q_C\) Critical value
\(Q_D\) Design value
\(Q_E\) Equilibrium value
\(Q_G\) Value for the gas
\(Q_{H1}\) Value at the inlet hub
\(Q_{H2}\) Value at the discharge hub
\(Q_i\) Components of vector \(Q\)
\(Q_i\) Pertaining to a section, \(i\), of the hydraulic system
\(Q_L\) Saturated liquid value
\(Q_m\) Meridional component
\(Q_M\) Mean or maximum value
\(Q_N\) Nominal conditions or pertaining to nuclei
\(Q_n, Q_t\) Components normal and tangential to whirl orbit
\(Q_P\) Pertaining to the pump
\(Q_r\) Component in the radial direction
\(Q_s\) Component in the \(s\) direction
\(Q_{T1}\) Value at the inlet tip
\(Q_{T2}\) Value at the discharge tip
\(Q_V\) Saturated vapor value
\(Q_x, Q_y\) Components in the \(x\) and \(y\) directions
\(Q_\theta\) Component in the circumferential (or \(\theta\)) direction

Superscripts and other qualifiers

On any variable, \(Q\):

\(\bar{Q}\) Mean value of \(Q\) or complex conjugate of \(Q\)
\(\tilde{Q}\) Complex amplitude of \(Q\)
\(\dot{Q}\) Time derivative of \(Q\)
\(\ddot{Q}\) Second time derivative of \(Q\)
\(Q^*\) Rotordynamics: denotes dimensional \(Q\)
\(Re\{Q\}\) Real part of \(Q\)
\(Im\{Q\}\) Imaginary part of \(Q\)
Introduction

1.1 Subject

The subject of this monograph is the fluid dynamics of liquid turbomachines, particularly pumps. Rather than attempt a general treatise on turbomachines, we shall focus attention on those special problems and design issues associated with the flow of liquid through a rotating machine. There are two characteristics of a liquid that lead to these special problems, and cause a significantly different set of concerns than would occur in, say, a gas turbine. These are the potential for cavitation and the high density of liquids that enhances the possibility of damaging unsteady flows and forces.

1.2 Cavitation

The word cavitation refers to the formation of vapor bubbles in regions of low pressure within the flow field of a liquid. In some respects, cavitation is similar to boiling, except that the latter is generally considered to occur as a result of an increase of temperature rather than a decrease of pressure. This difference in the direction of the state change in the phase diagram is more significant than might, at first sight, be imagined. It is virtually impossible to cause any rapid uniform change in temperature throughout a finite volume of liquid. Rather, temperature change most often occurs by heat transfer through a solid boundary. Hence, the details of the boiling process generally embrace the detailed interaction of vapor bubbles with a solid surface, and the thermal boundary layer on that surface. On the other hand, a rapid, uniform change in pressure in a liquid is commonplace and, therefore, the details of the cavitation process may differ considerably from those that occur in boiling. Much more detail on the process of cavitation is included in later sections.

It is sufficient at this juncture to observe that cavitation is generally a malevolent process, and that the deleterious consequences can be divided into three categories. First, cavitation can cause damage to the material surfaces close to the area where the bubbles collapse when they are convected into regions of higher pressure. Cavitation damage can be very expensive, and very difficult to eliminate. For most designers
of hydraulic machinery, it is the preeminent problem associated with cavitation. Frequently, one begins with the objective of eliminating cavitation completely. However, there are many circumstances in which this proves to be impossible, and the effort must be redirected into minimizing the adverse consequences of the phenomenon.

The second adverse effect of cavitation is that the performance of the pump, or other hydraulic device, may be significantly degraded. In the case of pumps, there is generally a level of inlet pressure at which the performance will decline dramatically, a phenomenon termed cavitation breakdown. This adverse effect has naturally given rise to changes in the design of a pump so as to minimize the degradation of the performance; or, to put it another way, to optimize the performance in the presence of cavitation. One such design modification is the addition of a cavitating inducer upstream of the inlet to a centrifugal or mixed flow pump impeller. Another example is manifest in the blade profiles used for supercavitating propellers. These supercavitating hydrofoil sections have a sharp leading edge, and are shaped like curved wedges with a thick, blunt trailing edge.

The third adverse effect of cavitation is less well known, and is a consequence of the fact that cavitation affects not only the steady state fluid flow, but also the unsteady or dynamic response of the flow. This change in the dynamic performance leads to instabilities in the flow that do not occur in the absence of cavitation. Examples of these instabilities are “rotating cavitation,” which is somewhat similar to the phenomenon of rotating stall in a compressor, and “auto-oscillation,” which is somewhat similar to compressor surge. These instabilities can give rise to oscillating flow rates and pressures that can threaten the structural integrity of the pump or its inlet or discharge ducts. While a complete classification of the various types of unsteady flow arising from cavitation has yet to be constructed, we can, nevertheless, identify a number of specific types of instability, and these are reviewed in later chapters of this monograph.

1.3 Unsteady Flows

While it is true that cavitation introduces a special set of fluid-structure interaction issues, it is also true that there are many such unsteady flow problems which can arise even in the absence of cavitation. One reason these issues may be more critical in a liquid turbomachine is that the large density of a liquid implies much larger fluid dynamic forces. Typically, fluid dynamic forces scale like \( \rho \Omega^2 D^4 \) where \( \rho \) is the fluid density, and \( \Omega \) and \( D \) are the typical frequency of rotation and the typical length, such as the span or chord of the impeller blades or the diameter of the impeller. These forces are applied to blades whose typical thickness is denoted by \( \tau \). It follows that the typical structural stresses in the blades are given by \( \rho \Omega^2 D^4 / \tau^2 \), and, to minimize structural problems, this quantity will have an upper bound which will depend on the material. Clearly this limit will be more stringent when the density of the fluid is larger. In many pumps and liquid turbines it requires thicker blades (larger \( \tau \) ) than would be advisable from a purely hydrodynamic point of view.
1.4 Trends in Hydraulic Turbomachinery

This monograph presents a number of different unsteady flow problems that are of concern in the design of hydraulic pumps and turbines. For example, when a rotor blade passes through the wake of a stator blade (or vice versa), it will encounter an unsteady load which is endemic to all turbomachines. Recent investigations of these loads will be reviewed. This rotor-stator interaction problem is an example of a local unsteady flow phenomenon. There also exist global unsteady flow problems, such as the auto-oscillation problem mentioned earlier. Other global unsteady flow problems are caused by the fluid-induced radial loads on an impeller due to flow asymmetries, or the fluid-induced rotordynamic loads that may increase or decrease the critical whirling speeds of the shaft system. These last issues have only recently been addressed from a fundamental research perspective, and a summary of the conclusions is included in this monograph.

1.4 Trends in Hydraulic Turbomachinery

Though the constraints on a turbomachine design are as varied as the almost innumerable applications, there are a number of ubiquitous trends which allow us to draw some fairly general conclusions. To do so we make use of the affinity laws that are a consequence of dimensional analysis, and relate performance characteristics to the density of the fluid, \( \rho \), the typical rotational speed, \( \Omega \), and the typical diameter, \( D \), of the pump. Thus the volume flow rate through the pump, \( Q \), the total head rise across the pump, \( H \), the torque, \( T \), and the power absorbed by the pump, \( P \), will scale according to

\[
Q \propto \Omega D^3 \\
H \propto \Omega^2 D^2 \\
T \propto \rho D^5 \Omega^2 \\
P \propto \rho D^5 \Omega^3
\]  

(1.1) (1.2) (1.3) (1.4)

These simple relations allow basic scaling predictions and initial design estimates. Furthermore, they permit consideration of optimal characteristics, such as the power density which, according to the above, should scale like \( \rho D^2 \Omega^3 \).

One typical consideration arising out of the affinity laws relates to optimizing the design of a pump for a particular power level, \( P \), and a particular fluid, \( \rho \). This fixes the value of \( D^5 \Omega^3 \). If one wished to make the pump as small as possible (small \( D \)) to reduce weight (as is critical in the rocket engine context) or to reduce cost, this would dictate not only a higher rotational speed, \( \Omega \), but also a higher impeller tip speed, \( \Omega D/2 \). However, as we shall see in the next chapter, the propensity for cavitation increases as a parameter called the cavitation number decreases, and the cavitation number is inversely proportional to the square of the tip speed or \( \Omega^2 D^2/4 \). Consequently, the increase in tip speed suggested above could lead to a cavitation problem. Often,